Integrating Language and Mathematics: Principles and Tools for Elementary Teachers and Students

Promising Math Conference
Erikson Institute Early Math Collaborative
October 13-14, 2017

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Galina (Halla) Jmourko, M. S., NBCT, Prince George’s County Public Schools, MD
Welcome!

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ESOL Enrollment Data
as of August 28, 2017

TOTAL ESOL POPULATION:
24,445

ELEMENTARY
K-5
15,657

MIDDLE
6-8
3,329

HIGH
9-12
5,459

ESOL Student Data

LANGUAGES
~130

COUNTRIES
~125

COUNTRY OF BIRTH
UNITED STATES (61%)
EL SALVADOR (19%)
GUATEMALA (5%)
HONDURAS (4%)
AFGHANISTAN (1%)
Questions for the Early Childhood Community

Big Ideas of Early Mathematics with English Learners:

1. What do teachers of young English learners need to know?
2. What actions do teachers of young English learners need to undertake when teaching mathematics?
3. What professional development structures need to be put in place to support teachers of young English learners?
Session Agenda

- UMD-PGCPS District Partnership: PD Initiatives
- Integrating Language and Mathematics
  - Principles That Guide Our Collaborative Work
  - Classroom Applications: Instructional Tools
- Implications for Early Childhood
- Closure and Questions

Session Outcomes: Participants will leave with actionable takeaways for addressing ELs’ needs in early childhood mathematics.
The Changing Landscape for Teaching Mathematics

- Growing numbers of English learners in public schools
- CCSS: Mathematical Content AND Practices
- WIDA Language Development Standards
  - ESOL teachers support language development (L, S, R, W) in **ALL** content areas, including **mathematics**
- **Teachers of Mathematics = Teachers of Language of Mathematics**

Focus on promoting student discourse for both **mathematics AND language development!**
OVERLAPPING KNOWLEDGE FOR TEACHING MATHEMATICS TO ENGLISH LANGUAGE LEARNERS

Discourse in Mathematics & the Language of Mathematics

Teaching and Learning of Mathematics - Content & Pedagogy

English Language Acquisition & English Learners

Mathematics Education/CCSS

WIDA ELD Standards
PGCPS & UMD Partnership

2014-15
FOCUS GROUP
Engaging ELLs in Mathematical Discourse*

GRAD COURSES
(ITQ Grant): ELL Math (ES)

2015-16
FOCUS GROUP
Supporting ELLs in Mathematics

GRAD COURSES
(ITQ Grant): ELL Math South

2016-17
FOCUS GROUP
Mathematical Literacy for ELLs*

PLC
ESOL Office-Based (Teaching Teams)

2017-18
FOCUS GROUP
Rigorous Math Instruction for ELLs (with County Mathematics Department)

GRAD COURSES
(ITQ Grant): ELGebra (MS)**

PLCs
School-Based (Teacher-Facilitated)

ESOL Coaching Support
Principles That Guide Our Collaborative Work

Discourse in Mathematics & the Language of Mathematics

Mathematics Education/CCSS

Teaching and Learning of Mathematics - Content & Pedagogy

WIDA ELD Standards

English Language Acquisition & English Learners

Recurring Question: How does this inform early childhood mathematics with English learners?
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What Does This Look Like in Early Childhood?
NCTM & NAEYC Position Statement

EXECUTIVE SUMMARY

Early Childhood Mathematics: Promoting Good Beginnings
A joint position of the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM)

Position
The National Council of Teachers of Mathematics and the National Association for the Education of Young Children (NAEYC) believe that early childhood education is a critical foundation for future mathematical learning. In early childhood settings, children should experience effective, research-based curriculum and teaching practices.

Position Statement
The NCTM and NAEYC believe that early childhood education is a critical foundation for future mathematical learning. In early childhood settings, children should experience effective, research-based curriculum and teaching practices.

Reflection/Discussion Prompt

- Hopefully this isn’t new to you and it is already informing early childhood instruction
- Important to consider implications for ELs

- Which of these 10 recommendations should be highlighted for ELs?
- Where should we place emphasis when “promoting good beginnings” for ELs?
English Language Development Standards

Standard 1: Social and Instructional Language
Standard 2: The Language of Language Arts
Standard 3: **The Language of Mathematics**
Standard 4: The Language of Social Studies
Standard 5: The Language of Science

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of mathematics.
Early English Language Development Standards

Early Years: 2.5 - 4.5

Standard 1: Social and Emotional Language
Standard 2: The Language of Early Language Development & Literacy
Standard 3: The Language of Mathematics
Standard 4: The Language of Social Studies
Standard 5: The Language of Science
Standard 6: The Language of Physical Development

www.wida.us
The Language of Mathematics
Defining Features (PreK-12)

Vocabulary:
everyday words
academic words
mathematics terms

Language Forms & Conventions:
language structures,
singular & plural nouns,
parts of speech, syntax

Linguistic Complexity:
extended & logically connected utterances
of speech (oral & written)

Word/Phrase Level
Sentence Level
Discourse Level

Listening, Speaking, Reading, Writing

Modified by Galina (Halla) Jmourko from The Defining Features of the Academic Language in WIDA’s Standards, WIDA Consortium, Draft, 2011
The Language of Mathematics
Defining Features (*Early Years: 2.5-4.5*)

Language Usage:
*children’s* developing sense of semantics, syntax, & pragmatics

Linguistic Complexity:
*children’s* variety & length of utterances

Modified by Galina (Halla) Jmourko from *The Performance Definitions of The WIDA Early English Language Development Standards, Ages 2.5–5.5, 2014 Edition*
Guiding Principles of Language Development (LD)

First languages, Cultures, Life Experiences

Guiding Principles of Language Development (LD)

1. Students' languages and cultures are valuable resources to be tapped and incorporated into instruction.
2. Students' home, school, and community experiences influence their language development.
3. Students draw on their metacognitive, multilingual, and multicultural awareness to develop proficiency in additional languages.
4. Students' academic language development in their native language facilitates their academic language development in English. Conversely, students' academic language development in English informs their academic language development in their native language.
5. Students learn language and culture through meaningful use and interaction.
6. Students use language in functional and communicative ways that vary according to context.
7. Students develop language proficiency in listening, speaking, reading, and writing independently, but at different rates and in different ways.
8. Students' development of academic language and academic content knowledge are inter-related processes.
9. Students develop socially, instructionally, and academically, a complex and long-term process.
10. Students' access to instructional tasks requiring complex thinking is enhanced when linguistic complexity and instructional support match their levels of language proficiency.

Listening, Speaking, Reading, & Writing

Social, Instructional, & Academic Language

Meaningful Interaction

Language - Content Connection

Supports

Context

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Early Years Guiding Principles of Language Development (LD)

Reflective Questions:

- How do these documents affirm and challenge your approaches/practices in the early childhood classroom?
- What 2-3 Principles will guide your work moving forward? Why?
Early Years Guiding Principles of Language Development (LD)

Key Points:

- Developmental considerations
- Simultaneous learning of home language (L1) AND of English as a new language (ENL)
- Role of (L1), culture, and experiences as valuable resources
- Different language learning paths of L1 and of ENL
- Role of play and meaningful interaction
What Does Integrated Instruction Look Like?

- **Discourse in Mathematics & the Language of Mathematics**
- **English Language Acquisition & English Learners**
- **Teaching & Learning of Mathematics - Content & Pedagogy**
- **Mathematics Education/CCSS**
- **NAEYC & NCTM**
- **WIDA ELD Standards**
- **WIDA Early ELD Standards**
## Guiding Principles of Language Development (LD)

### First languages, Cultures, Life Experiences

1. Students’ languages and cultures are valuable resources to be tapped and incorporated into instructional contexts.
2. Students’ home, school, and community languages influence their language development.

### Context

1. Students learn language through meaningful use and interaction.
2. Students use language in functional and communicative ways that vary according to context.
3. Students develop language proficiency in listening, speaking, reading, and writing.
4. Students develop semantic and academic language and academic content knowledge are interrelated processes.
5. Students develop social, instructional, and academic language, a complex and long-term process that is the foundation for their success in school.

### Language - Content Connection

10. Students’ access to instructional tasks requires complex thinking and instructional support that matches their levels of language proficiency.

### Meaningful Interaction

1. Students’ languages and cultures are valuable resources to be tapped and incorporated into instructional contexts.
2. Students use language in functional and communicative ways that vary according to context.
3. Students develop language proficiency in listening, speaking, reading, and writing.
4. Students develop semantic and academic language and academic content knowledge are interrelated processes.

### Social, Instructional, & Academic Language

10. Students’ access to instructional tasks requires complex thinking and instructional support that matches their levels of language proficiency.

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Integrated Approach to Mathematics - Language Teaching and Learning

Isolated Approach:
- Addressing ELs’ areas of needs in isolation
- Limited opportunities to practice language & math skills
- Limited professional collaboration

Integrated Approach: Educator Level
- Exploring & applying interrelated nature of math & language learning
- Building on teachers’ professional expertise
- Increasing opportunities for collaboration

Integrated Approach: Student Level
- Connecting to ELs’ backgrounds & experiences
- Math sense-making through language & non-linguistic representations
- Advancing language skills through interaction with math content & other students/adults

Shared responsibility for ELs’ learning experiences in math
# Language Development Supports for ELLs to Increase Comprehension and Communication

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<th>Environment</th>
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<td>Welcoming and stress-free</td>
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<tr>
<td>Respectful of linguistic and cultural diversity</td>
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<tr>
<td>Honors students’ background knowledge</td>
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<tr>
<td>Sets clear and high expectations</td>
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<tr>
<td>Includes routines and norms</td>
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<tr>
<td>Is thinking-focused vs. answer-seeking</td>
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<td>Offers multiple modalities to engage in content learning and to demonstrate understanding</td>
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<tr>
<td>Includes explicit instruction of specific language targets</td>
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<tr>
<td>Provides participation techniques to include all learners</td>
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<tr>
<td>Integrated learning center and games in a meaningful way</td>
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<tr>
<td>Provides opportunities to practice and refine receptive and productive skills in English as a new language</td>
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<tr>
<td>Integrates meaningful and purposeful tasks/activities that</td>
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<tr>
<td>Are accessible by all students through multiple entry points</td>
</tr>
<tr>
<td>Are relevant to students’ lives and cultural experiences</td>
</tr>
<tr>
<td>Build on prior mathematical learning</td>
</tr>
<tr>
<td>Demonstrate high cognitive demand</td>
</tr>
<tr>
<td>Offer multiple strategies for solutions</td>
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<tr>
<td>Allow for a language learning experience in addition to content</td>
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## Sensory Supports*
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*
- Graphs
- Charts
- Timelines
- Numberlines
- Graphic organizers
- Graphing paper

## Interactive Supports*
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports
- Labeling
- Students’ native language
- Modeling
- Repetition
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time
Exploration of Instructional Tools

Cubing Game

Three-Way Tie

2x2 Sentence Builders

Problem-Solution Space
Cubing Game: Looking at a Concept from Different Perspectives

Define

Compare/Contrast

Describe

Connect/Associate

Apply
Three Way Tie

Let’s Investigate And Prove The Connections!

Area measures inside the measure of the square. To find the area, you can count inside the square and you can find the area.

The perimeter is when you add up all the edges.

\[ A = 30 \quad P = 21 \]
2x2 or 3x3 Sentence Builders

- Fraction
- Numerator
- Equally

- Denominator
- Decimal Fraction
- Parts

- Convert
- Number
- The Whole
We are learning about 3 dimensional shapes. Here are the words I know and use when I talk about these shapes:

I can use these words to create at least 3 different sentences.

1. The Cube and the cone are different because the Cube has six faces and the Cone has two faces.
2. The Cube and the Cone are the same because the Cube and Cone are 3 dimensional shapes.
3. The Cone and the Cube are the same because the Cone has edges and the Cube has faces.
4. Also, 

---

Cube

Cylinder

Cone

Sphere
# My Problem-Solution Space

I am learning how to **make sense of a math problem** and how to **make a convincing argument** about my solution.

- **Paraphrase** or retell the problem in your own words.
- Create and label a **visual model to represent** the problem and the solution.

- **Use numbers** to solve the problem.

- Write your **answer** in complete sentences.
- Use specific **information or examples** from the problem to support your answer.
- Apply what you know mathematically to **make a convincing argument about your solution**.

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Problem – Solution Space

Paraphrase or restate the math problem in your own words.
Sketch the problem and the solution.

How many cardinals there is going be in saturday.
2, 5, 4, 7, 6, 9

Rule: add 3 subtract 1!
Ms. PB: “I didn’t know my kids can talk like that!”
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Instructional Tools: Take-Aways

• Have explicit language focus (vocabulary, sentences) AND support development of mathematical ideas
• Integrate multiple ways to support language and mathematics learning (visual, graphic, interactive)
• Offer ways to differentiate support for ELs of varying language skills (word level, sentence level, discourse level)
• Offer multiple ways for formative assessments (language AND mathematics)
• Create opportunities to practice CC Mathematical Practices AND L/S/R/W

NOTE: Because these are “non-traditional” ways of engaging students in mathematics, it is important to strategically coordinate the use of Tools with appropriate mathematical Tasks.
What Does It Look Like in the Early Childhood Classroom with Young ELs?

Considering NAEYC, NCTM, & WIDA, reflect on the following questions:

1. How might these **instructional tools** - that promote mathematics and language integration and foster literacy and discourse - look in the early childhood classroom?

2. What specific **modifications** might be necessary to support young ELs in the early childhood classroom?

3. What early childhood classroom **experiences** would prepare ELs for the type of integrated elementary instruction we have described?
Mathematics - English Learners - Early Childhood

Our Take-Aways

• Language-Rich Environment
  • Teachers’ attention to students’ receptive understanding
  • Teachers’ attention to nonverbal cues of students’ thinking
  • Some explicit instruction targeting specific language features

• Integrating Mathematics into Activities and Play
  • Not always “Math Block”

• Context and Shared Experiences
  • Literature, play, manipulatives, classroom routines, school events

• Developmentally Appropriate Instructional Tools
  • Reading/Writing → Representing/Modeling (not just numbers)
  • Translation/Cognates → Concept Attainment
Questions for the
Early Childhood Community

Big Ideas of Early Mathematics with English Learners:

1. What do teachers of young English learners need to know?
2. What actions do teachers of young English learners need to undertake when teaching mathematics?
3. What professional development structures need to be put in place to support teachers of young English learners?
Resources

- TODOS-math.org
- NCTM:
  - Principles to Actions
  - Access and Equity: Promoting High Quality Mathematics
  - Beyond Good Teaching
- TESOL
- Intentional Talk
THANK YOU!

Feel free to contact us for more information, resources, etc.

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